Integration Bee

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2 hours 30 minutes

Name: _____ Date: _____

Instructions:

- This paper is a **non-calculator** paper. You are **not** allowed to use any electrical devices.
- Each question is worth 5 marks.
- You do not need to concern yourself with issues of convergence.
- You must simplify your final answer.
- Partial marks will be awarded to answers which are incomplete, yet exhibit a method which is known to produce the correct answer, either by triviality, or through a complete solution using said method by another contestant.
- With this booklet, you will also be supplied an **answer booklet**. Although all submitted scrap working will be marked, you are required to list all stages (e.g "substitution of..." or "DUTIS on the function...") of your working in at most 5 steps. This is to assist in the handing of partial marks

Information:

- This paper consists of 3 pages.
- The total mark for this paper is 100.

1.

$$\int (x - x\ln(x))^{-1}\,\mathrm{d}x$$

2.

$$\int \operatorname{sech}(x)\,\mathrm{d}x$$

3.

$$\int \sec^5(x)\,\mathrm{d}x$$

4.

$$\int (1 + \ln(x))\ln(\ln(x))\,\mathrm{d}x$$

5. For any n,

$$\int x^{n-1}\arctan\left(\sqrt{x^n - 2}\right)\mathrm{d}x$$

6.

$$\int \ln\left(x + \frac{1}{x}\right)\mathrm{d}x$$

7. For a real constant y,

$$\int \frac{1}{x\ln(xy)\ln\left(\frac{x}{y}\right)}\,\mathrm{d}x$$

8.

$$\int_0^2 \ln^3\left(\frac{2}{x} - 1\right)\mathrm{d}x$$

9.

$$\int_0^\infty \frac{\lfloor x\rfloor^2}{\lfloor x\rfloor!}\cos(\pi\lfloor x\rfloor)\,\mathrm{d}x$$

10.

$$\int_0^\pi \cos(x + \tan(x))\,\mathrm{d}x$$

11.

$$\int \arctan\left(\frac{1}{\sqrt{2}}\left(\frac{1}{x}-\frac{x}{2}\right)\right)\mathrm{d}x$$

12.

$$\int \ln\left(1+x^4\right)$$

13.

$$\int \sec(x)\,\mathrm{arsinh}(\tan(x))\,\mathrm{d}x$$

14.

$$\int \frac{e^{-2\,\mathrm{artanh}(\tan(x))}}{\ln(1+\sin(2x))}\,\mathrm{d}x$$

15.

$$\int_2^\infty \frac{2}{\lfloor x\rfloor^2(\lfloor x\rfloor+2)}-\cosh^2(\mathrm{artanh}\lfloor x\rfloor)\,\mathrm{d}x$$

16.

$$\int_0^\infty \frac{e^{-x}}{x}\sin\left(\frac{x}{2}\right)\mathrm{d}x$$

17.

$$\int_0^{\frac{\pi}{2}} \tan(x)\arctan(\cos(x))\,\mathrm{d}x$$

18. For integer constants k and n such that $k>-1$ and $n+k<1$

$$\int_0^1 \frac{(2x)^k(1+x^2)^{n-1}}{(x^2-1)^{n+k}}\,\mathrm{d}x$$

19.

$$\int_1^\infty \frac{1}{x^2}\frac{2}{\pi}\arcsin\left|\sin\left(\frac{\pi}{2}x\right)\right|\mathrm{d}x$$

20. For a non-negative integer, n,

$$\int_1^e \frac{\ln(x)}{x}\ln^n(\ln(x))\,\mathrm{d}x$$